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Your Roll No.....

Sr. No. of Question Paper : 851

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Unique Paper Code : 2352572301

Name of the Paper : Differential Equations

Name of the Course : B.Sc. (Physical Science and
Mathematical Science) with
Operational Research and
Bachelor of Arts

Semester : III

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts form each question.
3. **All** questions carry equal marks.

1. (a) Find the general solution of the Bernoulli equation

given by $\frac{dy}{dx} + \frac{y}{2x} = \frac{x}{y^3}$ with initial condition $y(1) = 2$.

Also find an integrating factor for the linear differential equation

$$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}. \quad (7\frac{1}{2})$$

P.T.O.

(b) Find the general solution of the differential equation $(x^2 - 3y^2)dx + 2xy dy = 0$ by showing it's a homogeneous equation. Also show that $M(tx, ty) = tM(x, y)$ for $M = y + \sqrt{x^2 + y^2}$. (7½)

(c) Determine the most general $N(x, y)$ for the equation $(x^{-2}y^{-2} + xy^{-3})dx + N(x, y)dy = 0$ such that the equation is exact and solve the resulting exact equation. (7½)

2. (a) Assume that the population of a certain city increase at a rate proportional to the number of inhabitants at any time, if the population doubles in 40 years, in how many years will it triple? (7½)

(b) Show that the relation $x^2 + y^2 - 25 = 0$ is an implicit solution of the differential equation $x + y \frac{dy}{dx} = 0$ on the interval $-5 < x < 5$. Explain whether the relation $x^2 + y^2 + 25$ is also an implicit solution of $x + y \frac{dy}{dx} = 0$. (7½)

(c) Find the particular solution of the linear system that satisfies the stated initial conditions :

$$\frac{dy_1}{dt} = y_1 + y_2, \quad y_1(0) = 1$$

$$\frac{dy_2}{dt} = 4y_1 + y_2, \quad y_2(0) = 6. \quad (7\frac{1}{2})$$

3. (a) Solve the initial value problem :

$$x^2 y'' + 3xy' + y = 0, \quad y(1) = 4, \quad y'(1) = -1. \quad (7\frac{1}{2})$$

- (b) Find a homogeneous linear ordinary differential equation for which two functions x^{-3} and $x^{-3} \ln x$ ($x > 0$) are solutions. Show also linear independence by considering their Wronskian.

(7½)

- (c) Consider the initial value problem:

$$\frac{dy}{dx} = 1 + y^2, \quad y(0) = 0.$$

Examine the existence and uniqueness of solution in the rectangle: $|x| < 5, |y| < 3$.

(7½)

4. (a) Find a general solution of the following nonhomogeneous differential equation :

$$y'' + 4y' + 4y = e^{-2x} \sin 2x. \quad (7\frac{1}{2})$$

- (b) Use the method of undetermined coefficients to find the particular solution of the differential equation :

$$y'' - 2y' + y = x^2 + e^x. \quad (7\frac{1}{2})$$

P.T.O.

- (c) Use the method of variation of parameters to find a particular solution of the differential equation :
 $y'' + y = \tan x \sec x.$ (7½)
5. (a) Find the general solution of the equation.
 $(x - y)y^2u_x + (x - y)x^2u_y = (x^2 + y^2)u$ (7½)
- (b) Eliminate the constants a and b from the equation
 $2z = (ax + y)^2 + b$ (7½)
- (c) Solve the initial value problem :
 $u_t + uu_t = x, \quad u(x, 0) = 1$ (7½)
6. (a) Find the general solution of the linear partial differential equation.
 $x(y^2 - z^2)u_x + y(z^2 - x^2)u_y + z(x^2 - y^2)u_z = 0$ (7½)
- (b) Use $v = \ln u$ and $v = f(x) + g(y)$ to solve the equation.
 $x^2 u_x^2 + y^2 u_y^2 = u^2$ (7½)
- (c) Reduce the equation: $x^2 u_{xx} + 2x u_{xy} + y^2 u_{yy} = 0$
 to canonical form and hence find the general solution. (7½)